Taller 1:

$$\tau^* = \frac{\alpha+\gamma}{1+\gamma}$$

Cómo depende τ^* de α y de γ? — Elasticidad frisch de oferta laboral $\left(\frac{1}{\gamma}\right)$

$$\boxed{\frac{\partial \tau^*}{\partial \alpha} > 0} \qquad \underbrace{\frac{\partial \tau^*}{\partial \gamma} > 0}$$

$$\Rightarrow \frac{\partial \tau^*}{\partial \left(\frac{1}{\gamma}\right)} < 0 \longrightarrow$$ Entre más elástica sea la oferta laboral, más bajo es el τ^* óptimo.

$(1-\alpha)$ es la elasticidad de producción con respecto a la mano de obra.

Sostenibilidad fiscal con impuesto al ingreso:

$$\sum_{t=1}^{\infty} \frac{G_t}{(1+r_1^g)\cdots(1+r_{t-1}^g)} = \sum_{t=1}^{\infty} \frac{T_t}{(1+r_1^g)\cdots(1+r_{t-1}^g)}$$

$$T_t = \tau_t^y y_t \quad , \quad G_t = g_t y_t$$

$$\Leftrightarrow \boxed{\sum_{t=1}^{\infty} \beta^{t-1} \frac{(\tau_t^y - g_t)}{1-g_t} = 0}$$

Ej: Supongamos que $g_t = g$ $\forall t$.

- Inicialmente, el gobierno opera con presupuesto balanceado periodo a periodo: $G_t = T_t \Leftrightarrow \tau_t^y = g \quad \forall t$.
- Supongamos que el gobierno decide eliminar el impuesto al ingreso en el primer periodo: $\tau_1^y = 0$
- Cómo debe ser τ_t^y, $t \geq 2$, para que las finanzas públicas sean sostenibles?

$\tau_t^y = \tau'$ constante para $t \geq 2$.

$$\sum_{t=1}^{\infty} \beta^{t-1} \frac{\tau_t^y}{1-g_t} = \sum_{t=1}^{\infty} \beta^{t-1} \frac{g_t}{1-g_t}$$

$$\tau_1^y = 0 \ , \quad \tau_t^y = \tau' \quad \forall\, t \geq 2$$

$$\frac{\cancel{\tau_1^y}^{\,0}}{1-\cancel{g_1}\,g} + \beta \frac{\cancel{\tau_2^y}^{\,\tau'}}{1-\cancel{g_2}\,g} + \beta^2 \frac{\cancel{\tau_3^y}^{\,\tau'}}{1-\cancel{g_3}\,y} + \dots = \frac{\cancel{g_1}^{\,g}}{1-\cancel{g_1}\,g} + \beta \frac{g_2}{1-g_2} + \beta^2 \frac{g_3}{1-g_3} + \dots$$

$$\beta \frac{\tau'}{1-g} + \beta^2 \frac{\tau'}{1-g} + \beta^3 \frac{\tau'}{1-g} + \dots = \frac{g}{1-g} + \beta \frac{g}{1-g} + \beta^2 \frac{g}{1-g} + \dots$$

$$\frac{\tau'}{1-g}\left(\beta + \beta^2 + \beta^3 + \dots\right) = \frac{\tau'}{1-g} \sum_{t=1}^{\infty} \beta^t = \frac{\beta \tau'}{1-g} \underbrace{\sum_{t=1}^{\infty} \beta^{t-1}}_{=\frac{1}{1-\beta}}$$

$$\sum_{t=1}^{\infty} \alpha^{t-1} = \frac{1}{1-\alpha}$$

$$= \frac{\beta \tau'}{1-g} \cdot \frac{1}{1-\beta}$$

$$\frac{g}{1-g} + \beta \frac{g}{1-g} + \beta^2 \frac{g}{1-g} + \dots = \frac{g}{1-g} \sum_{t=1}^{\infty} \beta^{t-1} = \frac{g}{1-g} \cdot \frac{1}{1-\beta}$$

$$\Rightarrow \frac{\beta \tau'}{\cancel{(1-g)}\cancel{(1-\beta)}} = \frac{g}{\cancel{(1-g)}\cancel{(1-\beta)}} \quad \Rightarrow \boxed{\tau' = \frac{g}{\beta}} \quad (*)$$

$\frac{g}{\beta} > 0 \Rightarrow$ Si gobierno decide reducir en $t=1$ impuesto al ingreso, deberá aumentarlo de $t \geq 2$.

Cómo evoluciona la deuda del gobierno?

$$G_t - D_t = T_t - (1+r_{t-1}^g) D_{t-1} \ , \quad D_0 = 0$$

$$\Rightarrow G_1 - D_1 = T_1 \qquad G_1 = g y_1 \ , \quad T_1 = 0$$

$$\Rightarrow \boxed{D_1 = g y_1}$$

$$G_2 - D_2 = T_2 - \underbrace{(1+r_1^g)}\, D_1$$

$$1+r_{t-1}^g = \frac{C_t}{\beta C_{t-1}} = \frac{y_t(1-g_t)}{\beta y_{t-1}(1-g_{t-1})}$$

$$1+r_1^g = \frac{y_2(1-g_2)}{\beta y_1(1-g_1)}$$

$$\overset{g y_2}{\cancel{G_2}} - D_2 = T_2 - \frac{y_2 \cancel{(1-g_2)}}{\beta \cancel{y_1} \cancel{(1-g_1)}} \cdot g\, \cancel{y_1}$$

$$\Rightarrow D_2 = g y_2 - \tau' y_2 + \frac{y_2 g}{\beta} = g y_2 - \cancel{\frac{g}{\beta}} y_2 + \cancel{\frac{g}{\beta}} y_2$$

$$\Rightarrow \boxed{D_2 = g y_2}$$

$$\vdots$$

$$\boxed{D_t = g y_t}$$

- Deuda del gobierno es igual al gasto público en ese periodo.
- El recaudo se dedica exclusivamente a pagar la deuda del gobierno del periodo anterior.
- La restricción de no ponzi del gobierno se cumple:

$$\lim_{T\to\infty} \frac{D_T}{(1+r_1^g)\cdots(1+r_{T-1}^g)} = \lim_{T\to\infty} \frac{g\, y_T}{(1+r_1^g)\cdots(1+r_{T-1}^g)}$$

$$= \lim_{T\to\infty} g\, \beta^{T-1} y_1 = 0$$

Impuesto al consumo:

$$G_t = g_t y_t$$

$$T_t = \tau_t^c C_t \qquad C_t + G_t = y_t \quad \Rightarrow \quad C_t = y_t - G_t$$

$$C_t = y_t - g_t y_t$$

$$C_t = (1-g_t) y_t$$

$$\Rightarrow T_t = \tau_t^c (1-g_t) y_t$$

Si el gobierno tiene presupuesto balanceado periodo a periodo:

$$T_t = G_t \quad \Rightarrow \quad \tau_t^c (1-g_t) y_t = g_t y_t$$

$$\Rightarrow \boxed{\tau_t^c = \frac{g_t}{1-g_t}}$$

Para que finanzas públicas sean sostenibles:

$$\sum_{t=1}^{\infty} \frac{G_t}{(1+r_1^g)\cdots(1+r_{t-1}^g)} = \sum_{t=1}^{\infty} \frac{T_t}{(1+r_1^g)\cdots(1+r_{t-1}^g)}$$

$g_t y_t$ $\qquad$ $\tau_t^c (1-g_t) y_t$

$$\Leftrightarrow \sum_{t=1}^{\infty} \frac{\left(g_t - \tau_t^c (1-g_t)\right) y_t}{(1+r_1^g)\cdots(1+r_{t-1}^g)} = 0$$

$$1+r_t^g = \frac{C_{t+1}(1+\tau_{t+1}^c)}{\beta C_t (1+\tau_t^c)}$$

$C_{t+1} = (1-g_{t+1}) y_{t+1}$

$C_t = (1-g_t) y_t$

$$1+r_t^g = \frac{(1-g_{t+1}) y_{t+1} (1+\tau_{t+1}^c)}{\beta (1-g_t) y_t (1+\tau_t^c)}$$

$$(1+r_1^g)(1+r_2^g)\cdots(1+r_{t-1}^g) = \frac{\cancel{(1-g_2) y_2 (1+\tau_2^c)}}{\beta (1-g_1) y_1 (1+\tau_1^c)} \cdot \frac{\cancel{(1-g_3) y_3 (1+\tau_3^c)}}{\beta \cancel{(1-g_2) y_2 (1+\tau_2^c)}} \cdots \frac{(1-g_t) y_t (1+\tau_t^c)}{\beta \cancel{(1-g_{t-1}) y_{t-1} (1+\tau_{t-1}^c)}}$$

$$= \frac{(1-g_t) y_t (1+\tau_t^c)}{\beta^{t-1} (1-g_1) y_1 (1+\tau_1^c)}$$

$$\frac{y_t}{(1+r_1^g)\cdots(1+r_{t-1}^g)} = \beta^{t-1}\frac{(1-g_1)\,y_1\,(1+\tau_1^c)}{(1-g_t)(1+\tau_t^c)}$$

$$\sum_{t=1}^{\infty}\left(g_t-\tau_t^c(1-g_t)\right)\cdot\beta^{t-1}\frac{(1-g_1)\,y_1\,(1+\tau_1^c)}{(1-g_t)(1+\tau_t^c)}=0$$

$$\Leftrightarrow \boxed{\sum_{t=1}^{\infty}\beta^{t-1}\frac{\left(g_t-\tau_t^c(1-g_t)\right)}{(1-g_t)(1+\tau_t^c)}=0}$$

Ej: Supongamos $g_t = g$ $\forall t$.

Inicialmente el gobierno tiene presupuesto balanceado: $\tau_t^c = \frac{g}{1-g}$

El gobierno decide eliminar el impuesto en el periodo 1:

$\tau_1^c = 0$, $\tau_t^c = \tau'$, $t \geq 2$

¿Cuál debe ser τ' para que las finanzas públicas sean sostenibles?

$$\sum_{t=1}^{\infty}\frac{\beta^{t-1}g_t}{(1-g_t)(1+\tau_t^c)} = \sum_{t=1}^{\infty}\beta^{t-1}\frac{\tau_t^c\cancel{(1-g_t)}}{\cancel{(1-g_t)}(1+\tau_t^c)}$$

$$\frac{\cancelto{g}{g_1}}{(1-\cancelto{g}{g_1})(1+\cancelto{0}{\tau_1^c})}+\beta\frac{\cancelto{g}{g_2}}{(1-\cancelto{g}{g_2})(1+\cancelto{\tau'}{\tau_2^c})}+\beta^2\frac{g_3}{(1-g_3)(1+\tau_3^c)}+\ldots$$

$$=\frac{\cancelto{0}{\tau_1^c}}{1+\cancelto{0}{\tau_1^c}}+\beta\frac{\cancelto{\tau'}{\tau_2^c}}{1+\cancelto{\tau'}{\tau_2^c}}+\beta^2\frac{\cancelto{\tau'}{\tau_3^c}}{1+\cancelto{\tau'}{\tau_3^c}}+\ldots$$

$$\Leftrightarrow \frac{g}{(1-g)}+\beta\frac{g}{(1-g)(1+\tau')}+\beta^2\frac{g}{(1-g)(1+\tau')}+\ldots$$

$$=\beta\frac{\tau'}{1+\tau'}+\beta^2\frac{\tau'}{1+\tau'}+\ldots$$

$$\Leftrightarrow \frac{g}{1-g}+\frac{g}{(1-g)(1+\tau')}\left(\beta+\beta^2+\beta^3+\ldots\right)=\frac{\tau'}{1+\tau'}\left(\beta+\beta^2+\ldots\right)$$

$$\Leftrightarrow \frac{g}{1-g} + \frac{g\beta}{(1-g)(1+\tau')} \sum_{t=1}^{\infty} \beta^{t-1} = \frac{\tau'\beta}{1+\tau'} \sum_{t=1}^{\infty} \beta^{t-1}$$

$$\Leftrightarrow \frac{g}{1-g} + \frac{g\beta}{(1-g)(1+\tau')(1-\beta)} = \frac{\tau'\beta}{(1+\tau')(1-\beta)}$$

$$\frac{g(1+\tau')(1-\beta)}{\cancel{(1-g)(1+\tau')(1-\beta)}} + \frac{g\beta}{\cancel{(1-g)(1+\tau')(1-\beta)}} = \frac{(1-g)\tau'\beta}{\cancel{(1-g)(1+\tau')(1-\beta)}}$$

$$g(1+\tau')(1-\beta) + g\beta = (1-g)\tau'\beta$$

$$\vdots$$

$$\Rightarrow \boxed{\tau' = \frac{g/\beta}{1-g/\beta}} \qquad \tau' > \frac{g}{1-g}$$