Modelo con infinitos periodos:

Ej: $u(C_t) = \ln C_t$

$$\mathcal{L} = \sum_{t=1}^{\infty} \beta^{t-1} \ln C_t + \lambda\left(\sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0) - \sum_{t=1}^{\infty} P_t C_t \right)$$

$$\beta \frac{P_t}{P_{t+1}} = \frac{C_{t+1}}{C_t} \iff C_{t+1} = \beta(1+r_t) C_t$$

Cond. opt:

$$\boxed{\begin{aligned} \beta \frac{P_t}{P_{t+1}} &= \frac{C_{t+1}}{C_t} \quad \leftarrow \text{Euler} \\ \sum_{t=1}^{\infty} P_t C_t &= \sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0) \end{aligned}}$$

$$P_{t+1} C_{t+1} = \beta P_t C_t$$

$$P_t C_t = \beta P_{t-1} C_{t-1}$$

$$P_{t-1} C_{t-1} = \beta P_{t-2} C_{t-2}$$

$$\vdots$$

$$P_t C_t = \beta P_{t-1} C_{t-1} = \beta(\beta P_{t-2} C_{t-2}) = \beta(\beta(\beta P_{t-3} C_{t-3})) = \dots = \beta^{t-1} P_1 C_1$$

$$\Rightarrow P_t C_t = \beta^{t-1} C_1$$

$$\sum_{t=1}^{\infty} a^{t-1} = \frac{1}{1-a}, \quad |a|<1$$

$$\Rightarrow \sum_{t=1}^{\infty} P_t C_t = \sum_{t=1}^{\infty} \beta^{t-1} C_1 = C_1 \sum_{t=1}^{\infty} \beta^{t-1} = \frac{C_1}{1-\beta} = \sum_{t=1}^{\infty} P_t y_t + b_0(1+r_0)$$

$$\Rightarrow \boxed{C_1^* = (1-\beta)\left(\sum_{t=1}^{\infty} P_t y_t + (1+r_0) b_0 \right)}$$

$$C_{t+1} = \beta \left(\frac{P_t}{P_{t+1}} \right) C_t \Rightarrow \boxed{C_2^* = \beta \left(\frac{P_1}{P_2} \right) (1-\beta) \left(\sum_{t=1}^{\infty} P_t y_t + (1+r_0) b_0 \right)}$$

$$C_3^* = \beta \left(\frac{P_2}{P_3} \right) \beta \left(\frac{\cancel{P_1}}{\cancel{P_2}} \right)(1-\beta) \left(\sum_{t=1}^{\infty} P_t y_t + (1+r_0) b_0 \right)$$

$=1$

$$\vdots$$

$$\boxed{C_t^* = \frac{(1-\beta)\beta^{t-1}}{P_t} \left(\sum_{t=1}^{\infty} P_t y_t + (1+r_0) b_0 \right)}$$

$$c_1 + b_1 = y_1 + (1+r_0) b_0 \Rightarrow \boxed{b_1^* = y_1 + (1+r_0) b_0 - c_1^*}$$

$$b_2^* = y_2 + (1+r_1) b_1^* - c_2^*$$

$$\vdots$$

Ejemplo: choque transitorio en el primer periodo:

$$r_t = \rho \qquad b_0 = 0$$

$$y_t = \bar{y} \qquad \forall t > 1$$

$$y_1 = \bar{y} + \varepsilon$$

$$u(c_t) = \ln c_t \qquad \beta = \frac{1}{1+\rho} \Rightarrow \beta(1+r_t) = \frac{1+\rho}{1+\rho} = 1$$

$$c_{t+1} = \beta(1+r_t) c_t \qquad \Rightarrow \beta = \frac{1}{1+r_t}$$

$$\Rightarrow \boxed{c_{t+1} = c_t} \text{ — consumo constante.}$$

$$P_t = \frac{1}{(1+r_1)\cdots(1+r_{t-1})} = \underbrace{\left(\frac{1}{1+r_1}\right)}_{=\beta} \underbrace{\left(\frac{1}{1+r_2}\right)}_{=\beta} \cdots \underbrace{\left(\frac{1}{1+r_{t-1}}\right)}_{=\beta} = \beta^{t-1}$$

$$c_1^* = (1-\beta)\left(\sum_{t=1}^{\infty} P_t y_t + \cancel{(1+r_0)} b_0^{\,0}\right) = (1-\beta)\left(\sum_{t=1}^{\infty} P_t y_t\right)$$

$$= (1-\beta)\left(P_1 y_1 + P_2 y_2 + P_3 y_3 + \ldots\right)$$

$$= (1-\beta)\left(\bar{y} + \varepsilon + \beta \bar{y} + \beta^2 \bar{y} + \beta^3 \bar{y} + \ldots\right)$$

$$= (1-\beta)\left(\varepsilon + \bar{y}(1 + \beta + \beta^2 + \beta^3 + \ldots)\right)$$

$$= (1-\beta)\left(\varepsilon + \bar{y} \sum_{t=1}^{\infty} \beta^{t-1}\right) = (1-\beta)\left(\varepsilon + \frac{\bar{y}}{1-\beta}\right)$$

$$\Rightarrow \boxed{c_1^* = \bar{y} + (1-\beta)\varepsilon}$$

Si β es más alto $\Rightarrow$ el hogar consume menos de ese choque en el presente y lo ahorra para el futuro.

$$\boxed{c_t^* = \bar{y} + (1-\beta)\varepsilon}$$

$$b_1^* = y_1 + (1+r_0)\,b_0 - c_1^* = \cancel{\bar{y}} + \varepsilon - (\cancel{\bar{y}} + (1-\beta)\varepsilon)$$

$$= \varepsilon - (1-\beta)\varepsilon = \beta\varepsilon$$

$$b_2^* = y_2 + (1+r_1)\,b_1^* - c_2^* = \cancel{\bar{y}} + \underbrace{(1+\rho)\,\beta}_{\beta(1+\rho) = \frac{1+\rho}{1+\rho} = 1}\varepsilon - (\cancel{\bar{y}} + (1-\beta)\varepsilon)$$

$$= \varepsilon - (1-\beta)\varepsilon = \beta\varepsilon$$

$$\vdots$$

$$\boxed{b_t^* = \beta\varepsilon}$$

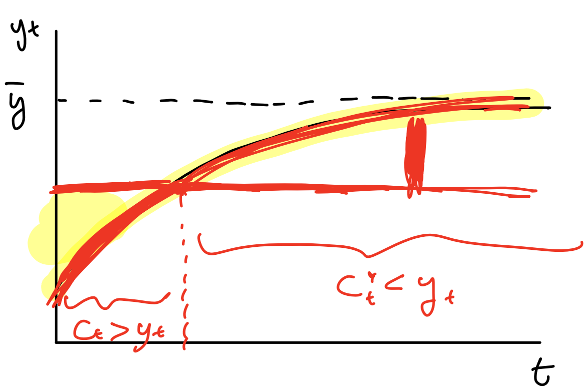
Hogar está ahorrando cada periodo $\beta\varepsilon$ y consume los intereses generados por ese ahorro cada periodo.

Ejemplo: dotaciones crecientes y convergentes:

$r_t = \rho$ $\qquad u(c_t) = \ln c_t$, $\qquad b_0 = 0$

$y_t = \bar{y} - \varepsilon^{t-1}$, $\qquad 0 < \varepsilon < 1$, $\qquad \bar{y} > 1$

$$\lim_{t\to\infty} y_t = \lim_{t\to\infty} \bar{y} - \varepsilon^{t-1} = \bar{y} - \lim_{t\to\infty} \varepsilon^{t-1} = \bar{y}$$



$r_t = \rho \Rightarrow \beta(1+r_t) = 1$

$\Rightarrow P_t = \beta^{t-1}$

$\Rightarrow c_{t+1} = c_t \longrightarrow$ consumo constante.

$$c_t^* = (1-\beta)\left(\sum_{t=1}^{\infty} P_t y_t\right) = (1-\beta)\left(P_1 y_1 + P_2 y_2 + P_3 y_3 + \dots\right)$$

$$= (1-\beta)\left(\bar{y} - 1 + \beta(\bar{y}-\varepsilon) + \beta^2(\bar{y}-\varepsilon^2) + \beta^3(\bar{y}-\varepsilon^3) + \dots\right)$$

$$= (1-\beta)\left(\bar{y} - 1 + \beta\bar{y} - \beta\varepsilon + \beta^2\bar{y} - \beta^2\varepsilon^2 + \beta^3\bar{y} - \beta^3\varepsilon^3 + \dots\right)$$

$$= (1-\beta)\Big(\bar{y}\underbrace{(1+\beta+\beta^2+\beta^3+\dots)}_{\sum_{t=1}^{\infty}\beta^{t-1}} - \underbrace{(1+\beta\varepsilon+(\beta\varepsilon)^2+(\beta\varepsilon)^3+\dots)}_{\sum_{t=1}^{\infty}(\beta\varepsilon)^{t-1}}\Big)$$

$$= (1-\beta)\left(\frac{\bar{y}}{1-\beta} - \frac{1}{1-\beta\varepsilon}\right)$$

$$\Rightarrow \boxed{c_t^* = \bar{y} - \frac{1-\beta}{1-\beta\varepsilon}}$$

$$b_1^* = y_1 + (1+r_0)b_0 - c_1^* = \cancel{\bar{y}} - 1 - \left(\cancel{\bar{y}} - \frac{1-\beta}{1-\beta\varepsilon}\right)$$

$$= \frac{1-\beta}{1-\beta\varepsilon} - 1 = \frac{\cancel{1} - \beta - \cancel{1} + \beta\varepsilon}{1-\beta\varepsilon}$$

$$= \frac{-\beta(1-\varepsilon)}{1-\beta\varepsilon} = \frac{\beta(\varepsilon-1)}{1-\beta\varepsilon} < 0$$

$$\boxed{b_1^* = -\beta\frac{(1-\varepsilon)}{1-\beta\varepsilon}}$$

$$b_2^* = y_2 + (1+r_1)b_1^* - c_i^* = \cancel{\bar{y}} - \varepsilon - \overbrace{(1+\rho)}^{=1}\beta\frac{(1-\varepsilon)}{1-\beta\varepsilon} - \left(\cancel{\bar{y}} - \frac{1-\beta}{1-\beta\varepsilon}\right)$$

$$= -\varepsilon - \frac{\beta(1-\varepsilon)}{1-\beta\varepsilon} + \frac{1-\beta}{1-\beta\varepsilon}$$

$$= \dots = -\beta\frac{(1-\varepsilon^2)}{1-\beta\varepsilon}$$

$$\boxed{b_2^* = -\beta\frac{(1-\varepsilon^2)}{1-\beta\varepsilon}} \qquad \boxed{b_3^* = -\beta\frac{(1-\varepsilon^3)}{1-\beta\varepsilon}} \qquad \dots \qquad \boxed{b_t^* = -\beta\frac{(1-\varepsilon^t)}{1-\beta\varepsilon}}$$

En cada periodo el hogar tiene deuda positiva.

$$\lim_{t\to\infty} b_t^* = \frac{-\beta}{1-\beta\varepsilon}$$

La condición de no-Ponzi se cumple:

$$\lim_{T\to\infty} \frac{b_T^*}{(1+r_1)\cdots(1+r_{t-1})} = \lim_{T\to\infty}\left(\frac{-\beta(1-\varepsilon^t)}{1-\beta\varepsilon}\right)\cdot\beta^{t-1} = 0$$

Equilibrio competitivo: consumos $\{c_t^i\}_{t=1}^{\infty}$, posiciones financieras $\{b_t^i\}_{t=1}^{\infty}$, tasas de interés $\{r_t\}_{t=1}^{\infty}$ y precios $\{p_t\}_{t=1}^{\infty}$ tal que:

1. Hogares escogen $\{c_t\}_{t=1}^{\infty}$, $\{b_t\}_{t=1}^{\infty}$ óptimamente dados $\{r_t\}_{t=1}^{\infty}$
2. mercados se vacían: $\sum_{i=1}^{I} c_t^{i*} = \sum_{i=1}^{I} y_t^i \quad \forall t$

$$\sum_{i=1}^{I} b_t^{i*} = 0 \qquad \forall t$$

Agente representativo: $b_t^{i*} = 0 \Rightarrow \boxed{c_t^* = y_t}$

$$\Rightarrow u'(c_t) = \beta(1+r_t)\,u'(c_{t+1})$$

$$\Rightarrow \boxed{1+r_t^* = \frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{u'(y_t)}{\beta u'(y_{t+1})}}$$ — tasas de interés de equilibrio.

Cobb-Douglas: $u(c_t) = \ln c_t$

$c_t^* =$ [redacted] $\left(\sum_{t=1}^{\infty} p_t y_t + (1+r_0) b_0\right)$

$$p_t = \frac{1}{(1+r_1)\cdots(1+r_{t-1})} = \underbrace{\left(\frac{1}{1+r_1}\right)}_{=\beta\frac{y_1}{y_2}}\cdots\underbrace{\left(\frac{1}{1+r_{t-1}}\right)}_{=\beta\frac{y_{t-1}}{y_t}}$$

$$= \left(\frac{\beta \cancel{y_{t-1}}}{y_t}\right)\cdot\left(\frac{\beta \cancel{y_{t-2}}}{\cancel{y_{t-1}}}\right)\cdots\left(\frac{\beta y_1}{\cancel{y_2}}\right) = \beta^{t-1}\frac{y_1}{y_t}$$

$$p_t = \frac{\beta^{t-1} y_1}{y_t}$$ → precios de equilibrio en econ. agente representativo.

$$\Rightarrow \sum_{t=1}^{\infty} p_t y_t = \sum_{t=1}^{\infty} \frac{\beta^{t-1} y_1}{\cancel{y_t}} \cdot \cancel{y_t} = y_1 \sum_{t=1}^{\infty} \beta^{t-1}$$

$$\Rightarrow \boxed{\sum_{t=1}^{\infty} p_t y_t = \frac{y_1}{1-\beta} < \infty}$$

Cuando utilidad es cobb-douglas, el problema del hogar siempre está bien definido.