Problem Set I

Dynamic Macroeconomics I

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ITAM

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1 Arrow-Debreu Equilibrium (30 points)

So far, we have assumed that markets are **sequential**, which means that every period there are markets for goods, factors of production, and savings. At every t, households receive wages w_t, returns to last period's capital $(1 - \delta + R_t)k_t$, and returns to savings $(1 + r_t)a_t$, and choose consumption c_t, capital k_{t+1}, and savings a_{t+1} for next period.

There is an equivalent way of studying markets, which will prove useful later in the course. Assume agents start life at period $t = 0$ and have perfect information about future income streams and prices in the economy. At $t = 0$, before any consumption is realized or any income received, agents meet at the market and trade *all* future consumption and capital accumulation streams, subject to a *lifetime* budget constraint. Once trade has occurred, agents own perfectly enforceable contracts every period on consumption and capital agreed in period $t = 0$.

Let p_t denote the price, in terms of consumption in period $t = 0$, of one unit of consumption to be delivered in period t (normalize $p_0 = 1$). Agents act competitively, which means that in period $t = 0$ they take future prices $\{p_t\}_{t=0}^{\infty}$ as given and beyond their control when making decisions.

The problem of the household is the following:

$$\max_{c_t, k_{t+1}} \quad \sum_{t=0}^{\infty} \beta^t u(c_t) \quad s.t.$$

$$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1}) = \sum_{t=0}^{\infty} p_t((1-\delta)k_t + w_t + R_t k_t)$$

$$c_t, k_{t+1} \geq 0, \quad k_0 \text{ given}$$

The equilibrium in this environment is called an **Arrow-Debreu (A-D) equilibrium**.

1. Solve for the A-D equilibrium
2. What is the relationship between prices p_t and the interest rate on risk-free bonds r_t?
3. Show the equivalence between the sequential markets and the A-D equilibria

   (*Hint:* use the no-Ponzi condition as stated in the class notes)

2 Transitional Dynamics (30 points)

Let an economy where the period utility is given by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and the production function is $F(k,l) = k^{\alpha} l^{1-\alpha}$. Assume that individuals can only invest in capital (i.e. there are no risk-free bonds) and let the parameters take the following values:

$$\beta = 0.99; \quad \sigma = 2; \quad \delta = 0.05; \quad \alpha = 0.3;$$

1. What is the value of steady state capital k^*, consumption c^*, investment i^*, production y^* and factor prices w^* and R^*?
2. Set $T = 10$ and compute the transitional dynamics using one of the methods studied in class. Assuming that the initial condition is $k_0 = 0.1 \cdot k_{ss}$, plot consumption c_t, investment i_t, production y_t, wages w_t, rental rate R_t, and the ratio $\frac{i_t}{y_t}$ on the path to steady state
3. How does the ratio $\frac{i_t}{y_t}$ compare to the Solow model?

3 Guess and Verify (20 points)

Let an economy where the period utility is given by $u(c) = \log(c)$, the production function is $F(k, l) = k^{\alpha} l^{1-\alpha}$, and there is full depreciation of capital $\delta = 1$. Guess that the value function takes the form:

$$v(k) = A + B \log(k)$$

In this environment, the social planner's problem can be written as:

$$v(k) = \max_{k'} \quad \log(k^{\alpha} - k') + \beta v(k')$$

Solve the social planner's problem to find the values of A and B, and find the **policy function** $k'(k)$. How does this model compare to the Solow model?

4 Value Function Iteration (20 points)

Using the same functional forms as in question 2, assume that the economy is in steady state. Solve the social planner's problem using the **value function iteration** method. Use a grid for capital of 500 equally spaced points, from $0.5k_{ss}$ to $1.5k_{ss}$. Plot the value function, and the capital and consumption policy functions.