

# Midterm Exam

## Dynamic Macroeconomics I

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### 1 Sequential Formulation (50 points)

Assume a representative agent economy with endogenous labor supply, no population growth ( $n = 0$ ), and no technological growth ( $g = 0$ ). Households value leisure and consumption, according to a lifetime utility function:  $\sum_{t=0}^{\infty} \beta^t u(c_t, 1-l_t)$ , where  $l_t$  is the amount of labor supplied. Capital depreciates at a constant rate  $\delta > 0$ , and there is a government that has expenditures equal to  $g_t$  at period  $t$ , where  $g_{t+1} = \rho g_t$ ,  $g_0$  is given and  $\rho \leq 1$ . The resource constraint is:

$$c_t + k_{t+1} + g_t = (1 - \delta)k_t + F(k_t, l_t)$$

Assume that households and the social planner take expenditures  $\{g_t\}_{t=0}^{\infty}$  as given and beyond their control.

- a. **Social planner:** State the social planner's problem sequentially and solve it to characterize the Pareto-optimal allocations.
- b. **Competitive equilibrium:** To finance expenditures  $g_t$ , assume there is a government that charges labor income taxes  $\tau_t^l$ , such that after-tax income of households is  $(1 - \tau_t^l)w_t l_t$ , where  $w_t$  is the wage. Also, the government levies consumption taxes  $\tau_t^c$ , such that total expenditures on consumption are equal to  $(1 + \tau_t^c)c_t$ . Note that  $\tau_t^c$  and  $\tau_t^l$  can be negative, in which case they are interpreted as subsidies.

Assume that every period the government chooses  $\tau_t^l$  and  $\tau_t^c$  such that its budget is

balanced in equilibrium, which means that:

$$g_t = \tau_t^l w_t l_t + \tau_t^c c_t$$

In addition, households have access to risk-free bonds that yield net returns equal to  $r_t$ . Define a competitive equilibrium.

- c. Solve the household's and firm's problem and characterize a competitive equilibrium.
- d. State the conditions under which there is a steady state and characterize steady state allocations.
- e. Is the competitive equilibrium Pareto optimal? You don't need to provide a formal proof, but use the answers above to explain your answer and give intuition. If the competitive equilibrium is not Pareto optimal, which policy could the government implement for the equilibrium to be Pareto optimal?

## 2 Recursive Formulation (30 points)

Assume the same environment as in Question 1.

- a. State the social planner's problem recursively. What are the state and control variables?
- b. Does a solution to the recursive formulation of the social planner's problem exist? Is it unique? Explain which conditions are sufficient to ensure the existence and uniqueness, and show that the social planner's problem satisfies these conditions.
- c. Define a recursive competitive equilibrium. Be sure to clearly identify what are the state and control variables of the household.