Final Exam

Dynamic Macroeconomics I

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ITAM

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1 Sequential Formulation (50 points)

Assume an Aiyagari-type heterogeneous agent economy in which there is a continuum of households, indexed by $i \in [0, 1]$. In period $t = 0$, agents choose the streams of consumption c_t and assets a_{t+1} that maximize their expected lifetime utility:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Every period, agents have one unit of time that they supply inelastically as labor, and face non-insurable idiosyncratic productivity shocks λ_t^i, such that their labor income is equal to $\lambda_t^i w_t$. Assume that $\lambda_t \in \Lambda$, and the possible set of realizations is finite: $|\Lambda| < \infty$. Households are ex-ante identical, but their histories $\lambda^{i,t} = (\lambda_1^i, \lambda_2^i, \ldots, \lambda_t^i)$ differ according to the realization of shocks. The probability that history $\lambda^{i,t}$ is realized is given by $\pi(\lambda^{i,t})$.

Assume also that markets are incomplete, so households only have access to a non-state contingent bond a_{t+1} that pays an interest r_t. Households choose a_{t+1} subject to an exogenous borrowing constraint $a_{t+1} \geq -\bar{A}$.

The final good is produced using capital K_t and labor L_t, according to a production function $F(K_t, L_t)$.

a. State the household's problem in sequential formulation and characterize the optimal allocations. In equilibrium, what are the optimal allocations functions of? *Briefly* discuss.

b. Assume now that markets are **complete**. What assets should be added to this environment? Write the household's problem and find the optimality conditions.

c. Are the equilibrium allocations of the complete and/or incomplete markets environments Pareto-optimal? *Briefly* explain.

d. Assume markets are incomplete and there is a government that observes the realization of histories $\lambda^{i,t}$ for all $i \in [0,1]$. The government can impose state contingent taxes $\tau(\lambda^{i,t})$ on asset returns. This means that the returns to a household with asset holdings a_t are $(1+(1-\tau(\lambda^{i,t}))r_t)a_t$. Can the government choose $\tau(\lambda^{i,t})$ such that the competitive equilibrium allocations are Pareto-optimal?

2 Recursive Formulation (30 points)

Assume the same environment as in Question 1.d., but now the productivity shocks λ_t^i follow a Markov process of order 1, where the transition probability is denoted as $\pi(\lambda_{t+1}^i|\lambda_t^i)$. In this environment, assume the government levies a tax on asset returns that only depends on the current realization of the shock: $\tau(\lambda_t^i)$.

a. State the household's problem in recursive formulation. What are the state variables?

b. Define a recursive competitive equilibrium.

c. Describe an algorithm to compute a *stationary* competitive equilibrium. (You don't have to describe how to solve the household's problem in here).

3 Wealth Inequality (20 points)

Wealth concentration has dramatically increased during the last decades in the U.S. In light of the papers studied in class, explain what are the main reasons for the increase in wealth concentration. What are the main ingredients of a model to study these mechanisms?