

# Wall Street or Main Street: Who to Bail Out?

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1. Question

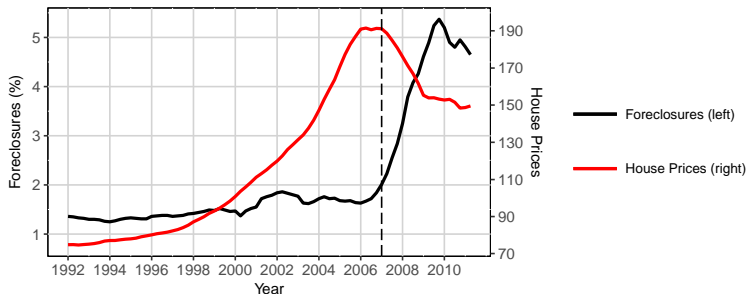
2. Quantitative Model

3. Baseline Calibration

4. One-Time Shock

# The Great Recession

- ▶ **Mortgage crisis:** decrease in house prices  $\Rightarrow$  increase in mortgage default/foreclosures



- ▶ Increase in foreclosures generated large losses to mortgage holders
- ▶ Threatened solvency of financial system

## Question

- ▶ What is the government policy that maximizes household welfare subject to preserving banks' solvency during mortgage crises?
  1. **Bailouts** to banks to cover losses
  2. **Subsidies to households** to prevent additional foreclosures
- ▶ Emergency Economic Stabilization Act - 2008:
  1. **Bailouts** (TARP): **\$60 billion (CBO)**
  2. **Subsidies to households** (HAMP): **\$75 billion**

# Trade-Off

The optimality of subsidies vs. bailouts is determined by 2 frictions:

1. Dead-weight loss on foreclosures of 20% (Campbell et al., 2011)
  - ▶ Prior to default households disinvest in the house
  - ▶ Vandalism and deterioration during vacancy
  - ▶ **Bailout policy** will pay dead-weight loss
2. Unobservable idiosyncratic house price component (10% – 14% std):
  - ▶ Government does not observe decision to default
  - ▶ **Subsidy policy**  $\Rightarrow$  additional 11% strategic default (Mayer et al., 2014)
  - ▶ If taxation is distortionary, this has welfare consequences

# I Abstract From...

- ▶ **Moral Hazard:**

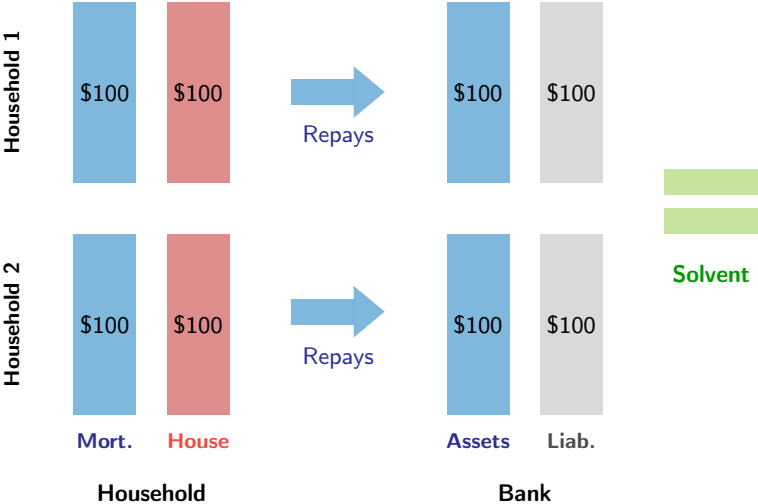
“Trying to mete out punishment to perpetrators ... by letting major firms (banks) fail ... can pour gasoline on the fire ... the truly moral thing to do during a raging financial inferno is to put it out.”

Tim Geithner (President of NY Fed, 2009):

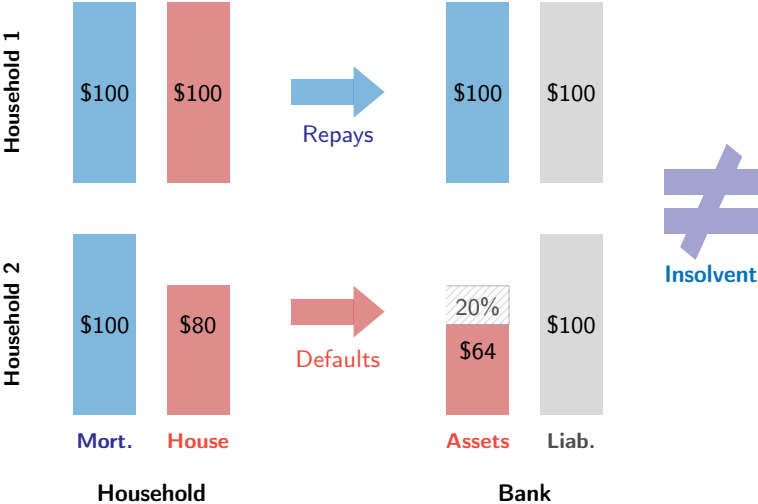
- ▶ **Price externality:**

- ▶ Effect seems low:  $-1\%$  on prices of houses  $< 0.1m$
- ▶ Including it reinforces results

# Trade-Off

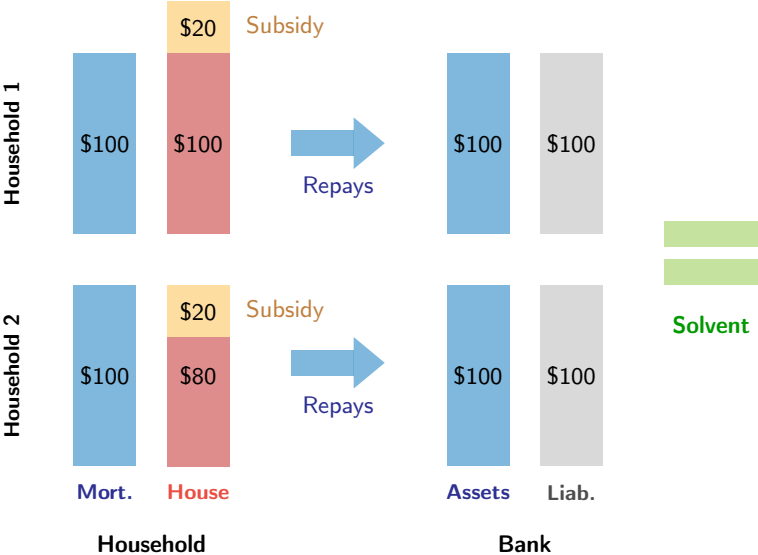


# Trade-Off

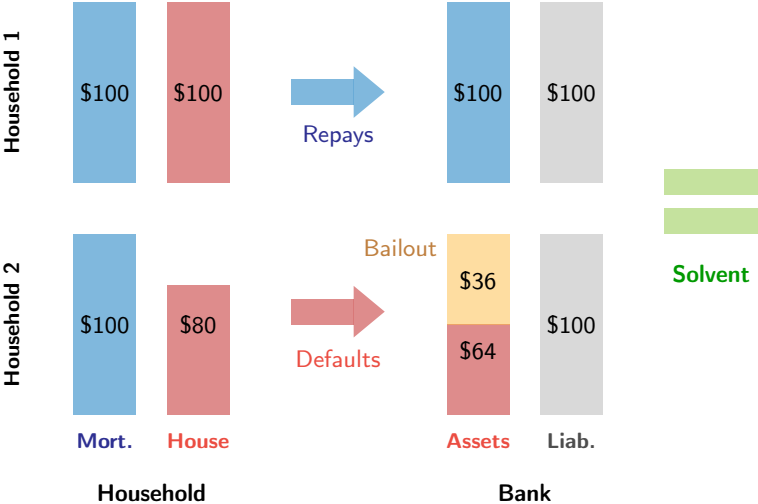




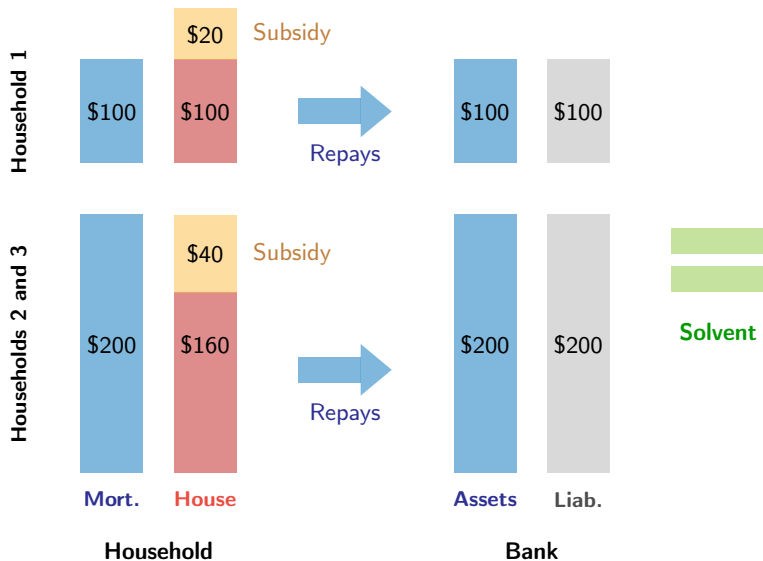
# Trade-Off



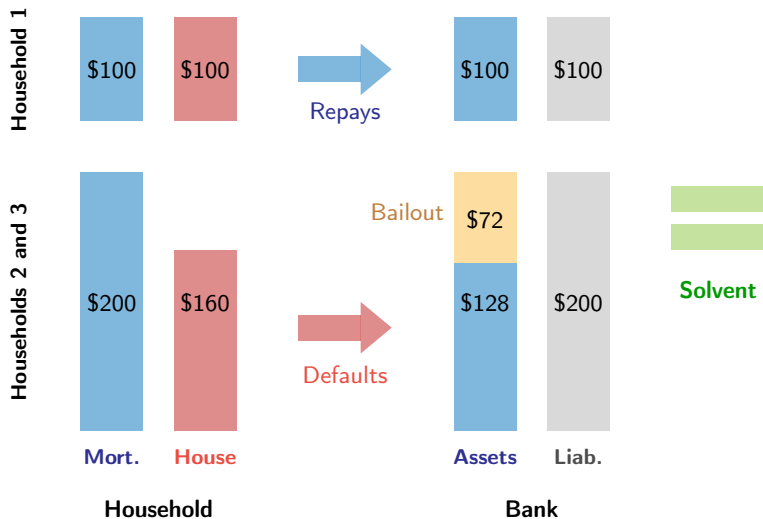
# Trade-Off



# Trade-Off



# Trade-Off



# Preview of Results

## 1. In the data:

- ▶ “Strategic default” is 11%:
  - ▶ Distortion given by Frisch elasticity is small
  - ▶ Not necessarily bad - can be welfare improving
- ▶ Dead-weight loss of 20% is large
- ▶  $\Rightarrow$  Subsidies outweigh bailouts!

## 2. Expanding HAMP to prevent *all* foreclosures and eliminating TARP:

- ▶ Welfare improvement of **+0.2%** in consumption terms

## 3. Implementing HAMP with better “eligibility” (first best):

- ▶ Welfare improvement of **+0.4%**

## 4. Expanding TARP and eliminating HAMP:

- ▶ Welfare improvement of **-0.8%**

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# Agents - Overview

## 1. Households

- ▶ Overlapping generations, heterogeneous agents
- ▶ Long term mortgages to finance housing

## 2. Mortgage originators

- ▶ Zero profits loan by loan

## 3. Production firms

- ▶ Linear technology, perfect competition

## 4. Government:

- ▶ In steady state: social security
- ▶ During crises: bailouts, mortgage subsidies and labor taxes
- ▶ Maximizes welfare, subject to ex-post solvency of banks

## Demographics + Preferences

- ▶ Overlapping generations, identical newborn households
- ▶ Live up to  $T = 30$  periods with exogenous survival probability  $\pi_t$
- ▶ Every period, cohort size  $\left(1 + \sum_{t=1}^T \prod_{i=1}^t \pi_i\right)^{-1}$  enters economy
- ▶ Lifetime utility:

$$\mathbb{E}_0 \sum_{j=1}^T \beta^{j-1} \pi_j \left[ \frac{\left(\psi \mathbf{c}_{t+j}^t{}^\kappa + (1 - \psi) \mathbf{s}_{t+j}^t{}^\kappa\right)^{\frac{1-\sigma}{\kappa}}}{1 - \sigma} - \theta_l \frac{\mathbf{l}_{t+j}^t{}^{1+\eta}}{1 + \eta} \right]$$

$\mathbf{c}_{t+j}^t$  : consumption,  $\mathbf{s}_{t+j}^t$  : housing services,  $\mathbf{l}_{t+j}^t$  : labor



# Income Dynamics

▶ Working age households:  $(1 - \tau_{ss}^j - \tau_G^j)we\bar{e}_t l$

1. After-tax market wage:  $(1 - \tau_{ss}^j - \tau_G^j)w$

2. Non-insurable idiosyncratic component:  $e$

$$\log(e') = \rho \log(e) + \sigma_\epsilon \epsilon, \quad \epsilon \sim N(0, 1)$$

3. Deterministic age-specific productivity:  $\bar{e}_t$

▶ Retired households:  $b$

▶ Owns production + mortgage origination firms (zero-profit)

# Housing

- ▶ Fixed supply  $\bar{H}$
- ▶ Own  $h$  at a price  $P_h^j$ :
  - ▶ Evolution:  $P_h^{j+1}(1 - \delta_i)h$      $\delta_i \sim U[-\underline{\delta}, \bar{\delta}]$ ,     $i \in [0, 1]$
  - ▶  $\delta_i$  partially observable:
    - ▶ Government observes  $\tilde{\delta}_i = \delta_i$  with probability  $1 - p$
    - ▶ Government observes  $\tilde{\delta}_i \in \Delta \setminus \{\delta_i\}$  with probability  $p$
  - ▶ Every period household hires labor to “reconstruct” depreciation:  $\delta_i h$
  - ▶ Production function of housing:  $f_h(L) = AL$
- ▶ Rent  $s$  at a price  $q^j$
- ▶ No owner occupied housing
- ▶ Can own and rent at same time (Jeske et al., 2013)
  - ▶  $h > s$ : net owner
  - ▶  $h < s$ : net renter

## Financial Assets

- ▶ One-period risk-free bonds:  $a' \geq 0$  at exogenous interest rate  $r_f^j$
- ▶ Housing is financed with mortgages  $m'$
- ▶ A mortgage  $m'$  with collateral  $h'$  and price  $P_m^j(t, e, a', h', m')$ :
  - ▶ Delivers  $P_m^j(t, e, a', h', m')m'$  on first period
  - ▶ Requires payments equal to  $m'$  every period
  - ▶ Debt disappears every period with probability  $\rho$
  - ▶ Proportional cost  $F$  on mortgage issuance/refinancing
  - ▶ Loan-to-value restriction at origination:

$$\left( \sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left[ \frac{\rho}{1+r} \right]^{j-t} \right) m' / P_h h' < LTV$$

- ▶ Default implies losing collateral  $h'$

# Household's Problem

- ▶ Heterogeneity across: age ( $t$ ), productivity ( $e$ ), savings ( $a$ ), housing ( $h$ ), mortgage debt ( $m$ ) and depreciation ( $\delta$ )
- ▶ State variables:  $\mathbf{s} := (t, e, a, h, m, \delta)$
- ▶ Choice variables: default/keep/refinance,  $c, h, s, a, l$
- ▶ Every period household solves:

$$V(\mathbf{s}) = \max_{c, s, h', a', m', l} \{V^{keep}(\mathbf{s}), V^{def}(\mathbf{s}), V^{ref}(\mathbf{s})\}$$

- ▶  $V^{keep}$  is value function for keeping current mortgage
- ▶  $V^{def}$  is value function for default
- ▶  $V^{ref}$  is value function for refinancing mortgage

Keep

Default

Refinance

# Firms

## 1. Production

- ▶ Linear technology:  $f(L) = A \cdot L$
- ▶  $w = A$
- ▶ Perfect competition

## 2. Mortgage originators

- ▶ Access to funds at equilibrium rate  $r_f$
- ▶ Perfect competition
- ▶  $P_m(j, e, a', H', m')$  determined by zero expected profit loan-by-loan

## Mortgage Pricing Function

$$P_m(t, e, a', h', m'; \Omega, \Theta) m' = \left( \frac{\pi_t \rho}{1 + r_f} \right) \mathbb{E}_{e', \delta'} \left[ \underbrace{d(s'; \Omega, \Theta) [(1 - \Psi) P_h (1 - \delta') h' - \delta' P_h h']}_{\text{Default}} + \right.$$

$$\underbrace{s(s'; \Omega, \Theta) \left( \sum_{j=t+1}^T [\Gamma_{i=t+1}^j \pi_i] \left( \frac{\rho}{1 + r} \right)^{j-(t+1)} \right) m'}_{\text{Refinance}} +$$

$$\left. \underbrace{(1 - d(s'; \Omega, \Theta))(1 - s(s'; \Omega, \Theta))(m' + P_m(t + 1, e', a'', h', m'; \Omega', \Theta') m')}_{\text{Keep mortgage}} \right]$$

## Government's Problem

- ▶ Government subsidizes mortgage refinancing at lump-sum  $\tau$ :

$$\tau : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \tilde{\Delta} \rightarrow [0, 1]$$

- ▶ Subsidy eligibility rule is:

$$\Gamma : \{1, \dots, T\} \times \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \tilde{\Delta} \rightarrow \{0, 1\}$$

- ▶ Chooses subsidies  $\tau(t, e, a, h, m, \tilde{\delta})$ , eligibility rule  $\Gamma(t, e, a, h, m, \tilde{\delta})$
- ▶ w.p.  $p$ , government gives subsidy to  $\Gamma(\mathbf{s}) = 0$  and  $\Gamma(\tilde{\mathbf{s}}) = 1$
- ▶ Mistakes only made on  $\delta$  but not on  $(t, e, a, h, m)$

# Government's Problem

- After ag. shock:    **subsidies**  $\tau$ ,    **bailouts**  $B$ ,    **labor income taxes**  $\tau_G$

$$\max_{\tau_M, \Gamma, B, \tau_G} \int_{s'} V(s'; \tau_M, \Gamma, B, \tau_G) d\Phi_j(s'), \quad \text{s.t.}$$

$$\int_{\mathbf{s}} P_m^{j-1}(t, e, a', h', m') m' \quad d\Phi_{j-1}(\mathbf{s}) = \quad \text{(Ex-Post Solv.)}$$

$$\left( \frac{\pi_j \rho}{1 + r_f^j} \right) \int_{s'} [d(s'; \tau_M, \Gamma, B, \tau_G) P_h(1 - \delta')(1 - \Psi) \mathbf{H}' + \dots] d\Phi_j(s') + B$$

$$(1 - \rho) \int_{s'} \Gamma(s') \tau(s') F(\dots) s(\cdot) d\Phi + \rho \int_{s'} (1 - \Gamma(s')) \tau(s') F(\dots) s(\cdot) d\Phi + \dots$$

$$\dots + B = A \quad \text{(Bud. Bal. } j \text{)}$$

$$Ar_f^i = \underbrace{\int \tau_G e \bar{e}_j w l(\cdot) d\Phi_i}_{\text{Taxes}} \quad \text{(Bud. Bal. } i > j \text{)}$$



# Equilibrium

A stationary recursive competitive equilibrium are a value function  $V$  and policy functions for household, a pricing function  $P_m(\cdot)$  for mortgages, a price for rental housing  $q$ , a price for new housing  $P_h$  and a distribution  $\Phi$  such that:

1. Households optimize Household's problem
2. Financial firms optimize (zero-profit condition for  $P_m$ ) Firms' problem
3. Markets clear Market clearing
  - 3.1 Rental housing
  - 3.2 Owned housing
  - 3.3 Bonds
  - 3.4 Goods
4. Government's budget balance Govs budget balance
5. Law of motion for aggregate distributions  $\Phi_t$

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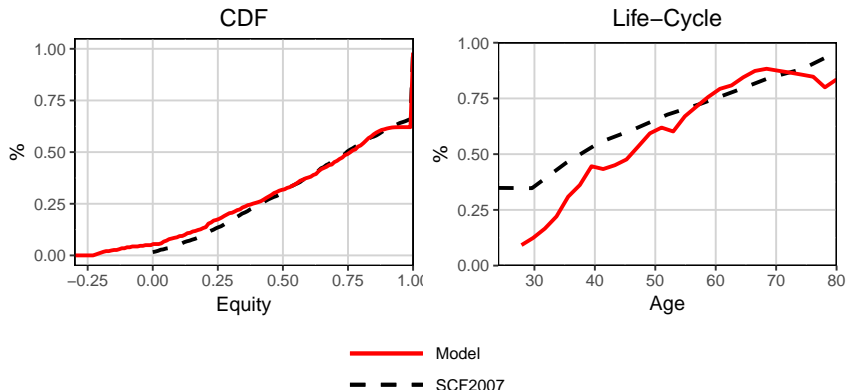
4. One-Time Shock

	Value	Source	Function
<b>Pref.</b>			
$\sigma$	2	IES: 0.5	$\frac{(\psi c^{\kappa} + (1-\psi)h^{\kappa})^{\frac{1-\sigma}{\kappa}}}{1-\sigma}$ $-\theta_l \frac{l^{1+\eta}}{1+\eta}$
$\kappa$	-0.1	Fernandez-Villaverde et al. (2011)	
$\eta$	0.5	Keane and Rogerson (2015)	
$\theta_l$	5	Average Labor = 0.4	
<b>Demog.</b>			
$T$	30	Maximum age: 80	
$\pi_t$	-	Actuarial Life Tables	
<b>Income</b>			
$\lambda_{\epsilon}$	0.95	Storesletten et al. (2004)	$\log(e_{t+1}) =$ $\lambda_{\epsilon} \log(e_t) + \sigma_{\epsilon} \epsilon_{t+1}$
$\sigma_{\epsilon}$	0.22	Storesletten et al. (2004)	
$\bar{e}_t$	-	Hansen (1993)	
<b>Financ.</b>			
$\rho$	0.92	Mortgage dur. 25 yrs	
$F$	0.015	Hurst and Stafford (2004)	
$\Psi$	0.22	Pennington-Cross (2006)	

# Endogenously Calibrated

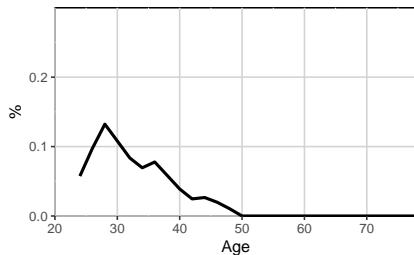
Parameter	Value	Variable	Data	Model
		<b>Targeted Moments</b>		
$\bar{\delta}$	0.119	Default rate	2.96%	2.5 %
$\underline{\delta}$	-0.345	Ownership rate	65.5%	65.6 %
$\psi$	0.84	Rent/Cons expenditures	14.1%	14.3 %
		<b>Untargeted Moments</b>		
		Std. Dev. Idiosyncratic Comp.	10 – 14.5%	13.4 %
		Average home equity	62%	64.3 %
		Average size (sq ft.): owned/rented	1.51	1.52

# Equity Distribution



- ▶ Misses left tail of distribution
- ▶ Agents start with zero assets + zero housing

## Default in Steady State



- ▶ Young households (25-35 yrs)
- ▶ Negative idiosyncratic price shock (high  $\delta$ )
  - ▶ Poor households that cannot pay
  - ▶ Wealthy households that can cover issuance cost of new mortgage later
- ▶ Only 31% of households underwater default:
  - ▶ Fixed cost of mortgage issuance  $F$
  - ▶ Fixed contracts  $\Rightarrow$  households in “bad shape” prefer to stick to contract

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# One-Time Shock

▶ Shocks last for three periods:

1. Loan-To-Value at origination set  $\leq 65\%$  - **Boz and Mendoza (2014)**  
(downpayment of 35%)

- ▶ Fraction of banks on *Willingness to Lend Survey* that tightened credit standards increased from 0% to 50%

2. Labor income shock - **Glover et al. (2014)**:

Age group	Per capita earnings
20 – 29	-12.8%
30 – 39	-11.1%
40 – 49	-8.8%
50 – 59	-9.6%
60 – 69	-4.4%
70+	+0.3%



# HAMP vs TARP: The Great Recession

- ▶ Eligibility conditions for HAMP:
  1. **Payments-to-income  $\geq 31\%$**
  2. **Delinquent or in danger of falling behind on mortgage payments**
  3. Collateral has to be owner-occupied and primary residence
  4. Single-family, 1-4 units, unmodified first-lien mortgage  $\leq \$730K$
  5. Originated before Jan. 1st, 2009
  6. Modification has to pass net present value test
- ▶ **HAMP reduced payments-to-income to exactly 31%** of monthly income
- ▶ Expenditures \$135:
  1. HAMP expected cost: \$75 billion (56%)
  2. TARP expected bailout: \$60 billion (44%)

## HAMP vs TARP: The Model

- ▶ Want to match policy that preserves solvency and transfers:
  1. 56% in mortgage refinancing subsidies (HAMP)
  2. 44% in bailouts to banks
- ▶ Such a policy subsidizes mortgage refinancing to:
  1. Households with payments-to-income  $\geq 28\%$
  2. Choose to default
- ▶ Subsidies such that PTI in period of subsidy is reduced to exactly 28%

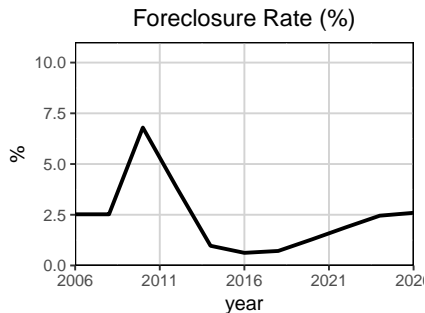
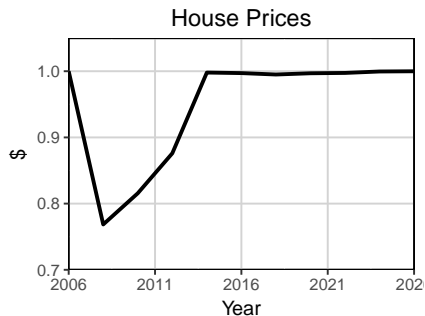
# Information Friction

- ▶ Mayer et al. (2014)
- ▶ Settlement between Countrywide Fin. Corp. and Federal Government
- ▶ Offered modifications to seriously delinquent, subprime, first lien mort.
- ▶ Find **strategic default** of 10 – 11%
- ▶ In equilibrium, government subsidizes 11% of non-defaulters with:

$$p = 1.6\%$$

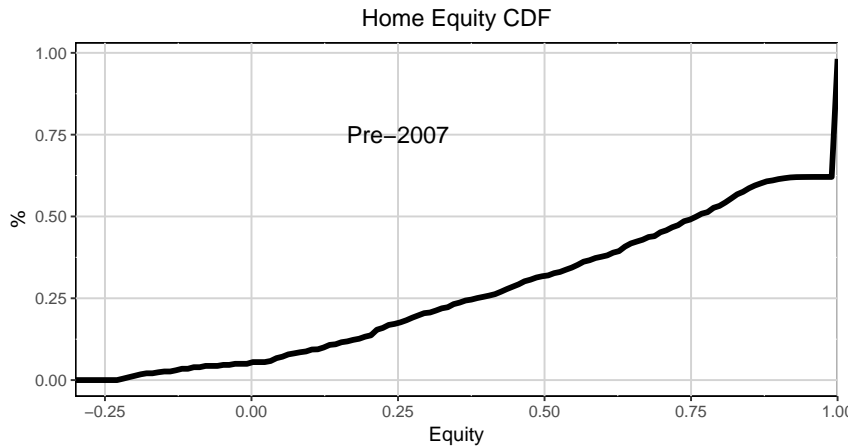
- ▶ Problem: point estimate!
- ▶ Upper bound - modifications were “unconstrained”

# The Great Recession

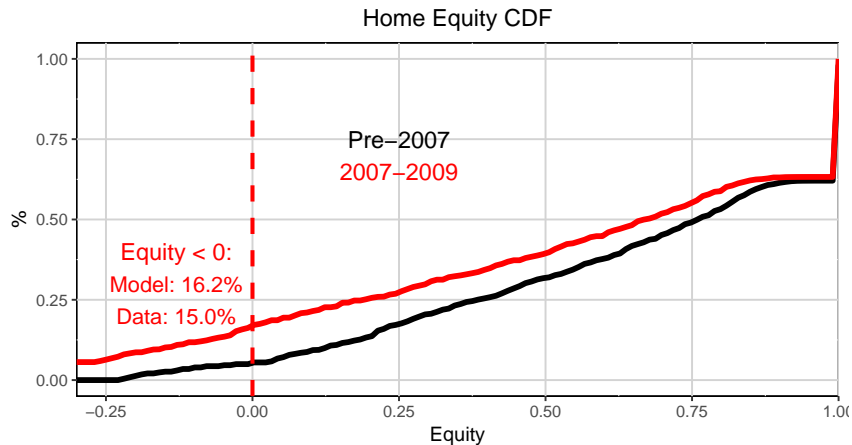


- ▶ Initially foreclosures rise
- ▶ “High-risk” households default while “low-risk” keep mortgage
- ▶ After income shock ends, better composition of mortgage holders

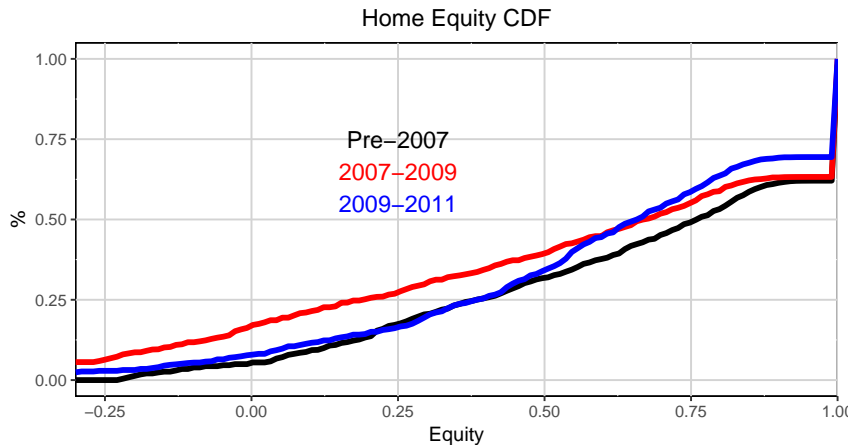
# The Great Recession



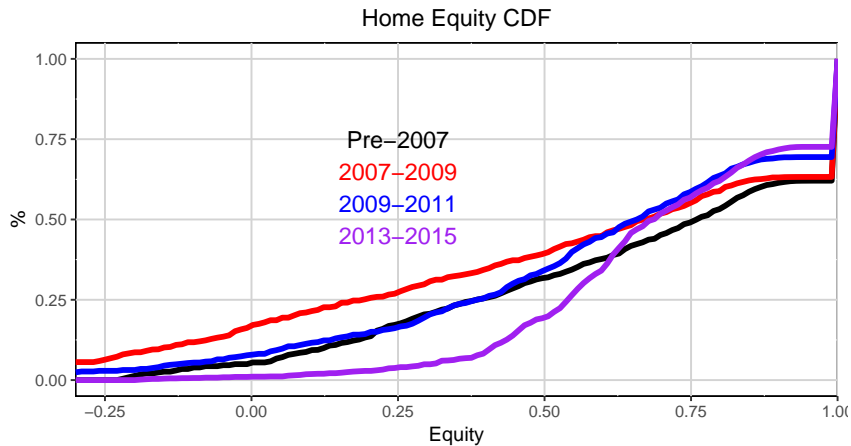
# The Great Recession



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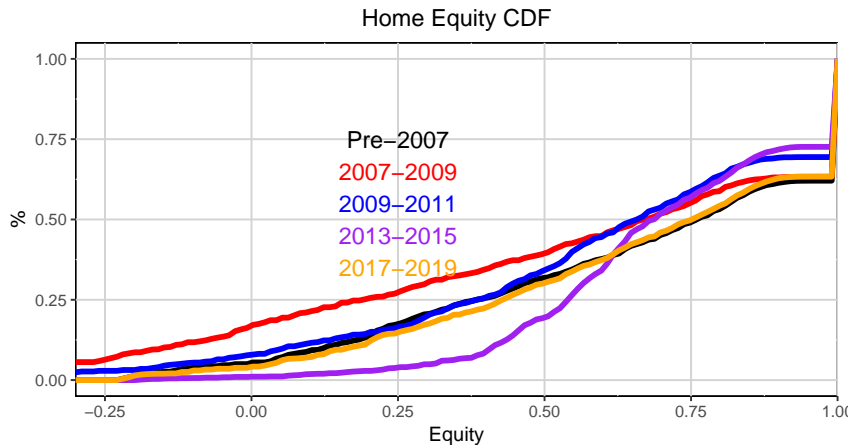


# The Great Recession

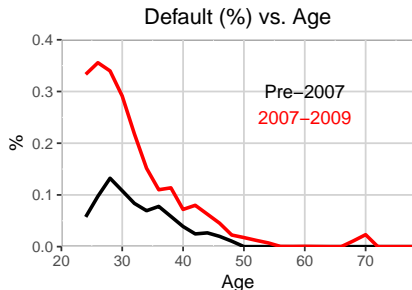
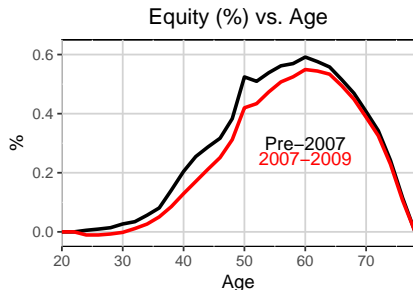




# The Great Recession



# The Great Recession



- ▶ Equity is low for young households
- ▶ ⇒ crisis hits hardest youngest home-owners
- ▶ Default concentrated on youngest

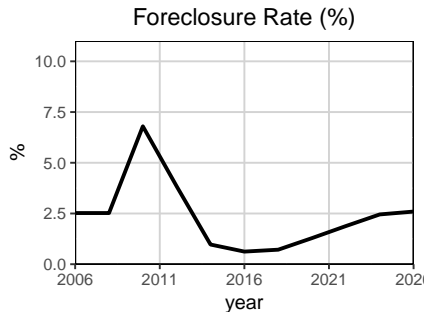
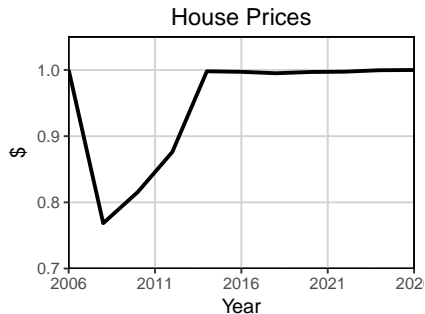
# Counter-Factual Policy #1

- ▶ First best policy:
  - ▶ Default after shock highly concentrated on young
  - ▶ Subsidize only households that changed default decision after shock
  - ▶ Subsidy amount is sum of two components:
    - ▶ Amount given under HAMP
    - ▶ Amount decreasing on age
  - ▶ Increases welfare by +0.4%
  - ▶ Decreases foreclosures to 4.9%
  - ▶ Small effect on house prices

## Counter-Factual Policy #2 and #3

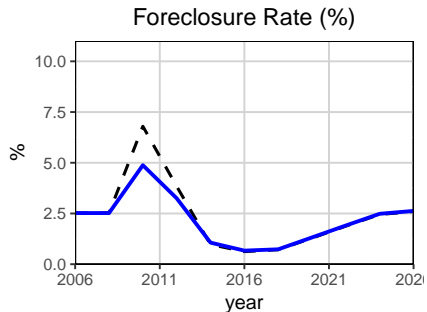
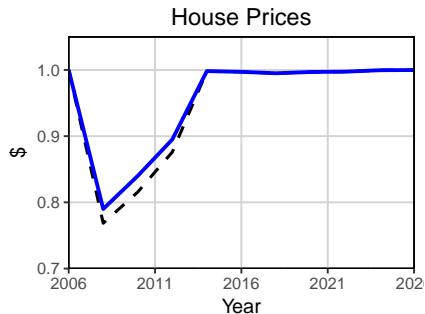
- ▶ Expand HAMP as large as possible: subsidize to  $PTI \geq 22.5\%$ :
  1. Increases welfare by 0.2%
  2. Has a negligible effect on prices
  3. Decreases default by 3% between 2007-2009
  
- ▶ Bailout-only policy:
  1. Decreases welfare by 0.8%
  2. Has a negligible effect on prices
  3. Increases default by 3.4% between 2007-2009

# The Great Recession



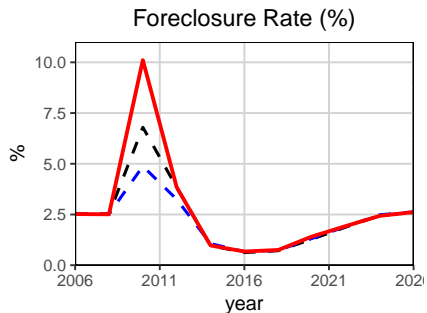
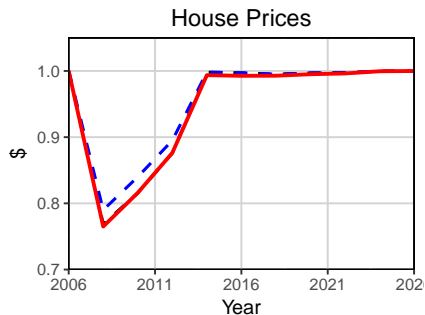
— Baseline Policy

# The Great Recession



- Baseline Policy
- Subsidy-only Policy

# The Great Recession



- Baseline Policy
- Subsidy-only Policy
- Bailout-only Policy

## Concluding Remarks

- ▶ Cost of dead-weight loss outweighs cost of information friction
- ▶  $\Rightarrow$  Better to prevent foreclosures
- ▶ Expanding subsidy policies would have generated welfare improvements over bailout policies of 0.2% – 0.4%
- ▶ Including foreclosure externality would make bailout policy even costlier
- ▶ Moral hazard can play in both directions, so effect is ambiguous



Thank you!

# Computation

1.  $\mathbf{t} = \mathbf{0}$  : Start with guess  $\mathbf{g}^0$  for policies and  $\mathbf{P}_m^0$  for pricing functions

2. **Iteration  $\mathbf{t}$**  :

2.1 Given  $\mathbf{g}^t$ , perform value function iteration of  $P_m$  until convergence (cont. map. thm) to get  $\mathbf{P}_m^{t'}$ . Set:

$$\mathbf{P}_m^{t+1} = \gamma \mathbf{P}_m^t + (1 - \gamma) \mathbf{P}_m^{t'}$$

2.2 Given  $\mathbf{P}_m^{t+1}$ , solve household's problem ( $t = T, \dots, 1$ ) to get  $\mathbf{g}^{t+1}$

2.3 If  $\|\mathbf{P}_m^{t+1} - \mathbf{P}_m^t\| < \epsilon$ , stop. Otherwise,  $\mathbf{t} = \mathbf{t} + 1$  and go to 2

3. If model and data moments match, stop. Otherwise, choose new parameters and repeat until convergence.

## Household's Problem: Keep Mortgage

$$V^{keep}(t, e, a, h, m, \delta; \Omega, \Theta) = \max_{\substack{c, s, h', a' \geq 0 \\ l \in [0, 1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e' | e, \delta', m'} V(t+1, e', a', h, m', \delta'; \Omega', \Theta')$$

$$c + m + qs + P_a a' + \frac{\delta h w}{A} = (1 - \tau_l - \tau_{ss}) e \bar{e}_t w l + a + qh$$

$$m' = \begin{cases} m & \text{w.p. } \rho \\ 0 & \text{w.p. } 1 - \rho \end{cases}$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e' | e), \quad \Omega' = G(\Omega), \quad V^{keep}(T+1, \cdot) = 0$$

Go to equilibrium

# Household's Problem: Default

$$V^{def}(t, e, a, h, m, \delta; \Omega, \Theta) = \max_{\substack{c, s, h', a', m' \geq 0 \\ l \in [0, 1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e' | e, \delta', m', \tilde{a}'} V(t+1, e', \tilde{a}', 0, 0, \delta'; \Omega', \Theta')$$

$$c + qs + P_a a' = (1 - \tau_l - \tau_{ss}) e \bar{e}_t w l + a$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e' | e), \quad \Omega' = G(\Omega), \quad V^{def}(T+1, \cdot) = 0$$

Go to equilibrium

## Household's Problem: Refinance

$$\mathbb{E}_p V^{ref}(t, e, a, h, m, \delta; \Omega, \Theta) = (1 - p)V^{ref, 1-p}(t, e, a, h, m, \delta; \Omega, \Theta) + pV^{ref, p}(t, e, a, h, m, \delta; \Omega, \Theta)$$

## Household's Problem: Refinance

$$V^{ref,1-p}(t, e, a, h, m, \delta; \cdot) = \max_{\substack{c, s, h', a', m' \geq 0 \\ l \in [0,1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e'|e, \delta', \tilde{m}'} V(t+1, e', a', h', \tilde{m}', \delta'; \cdot)$$

$$c + \left( \sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left( \frac{\rho}{1+r} \right)^{j-t} \right) (m + (1 - \Gamma(\cdot)\tau(\cdot))Fm') + qs + P_h h' + P_a a' = \\ (1 - \tau_l - \tau_{ss})e\bar{e}_t w l + a + P_h h - \frac{\delta P_h h w}{A} + qh' + P_m(t, e, a', h', m'; \cdot, \Theta)m'$$

$$\tilde{m}' = \begin{cases} m' & \text{w.p. } \rho \\ 0 & \text{w.p. } 1 - \rho \end{cases} \quad F = \begin{cases} F^{issue} & \text{if } m = 0, m' > 0 \\ F^{ref} & \text{if } m > 0, m' > 0 \\ 0 & \text{if } m' = 0 \end{cases}$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e'|e), \quad \Omega' = G(\Omega), \quad V^{ref}(T+1, \cdot) = 0$$

$$\left( \sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left[ \frac{\rho}{1+r} \right]^{j-t} \right) m' / P_h h' < LTV$$

## Household's Problem: Refinance

$$V^{ref, 1-p}(t, e, a, h, m, \delta; \cdot) = \max_{\substack{c, s, h', a', m' \geq 0 \\ l \in [0, 1]}} u(c, s) + \pi_t \beta \mathbb{E}_{e'|e, \delta', \tilde{m}'} V(t+1, e', a', h', \tilde{m}', \delta'; \cdot),$$

$$c + \left( \sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left( \frac{\rho}{1+r} \right)^{j-t} \right) (m + (1 - (1 - \Gamma(\cdot))\tau(\cdot))Fm') + qs + P_h h' + P_a a' = \\ (1 - \tau_l - \tau_{ss})e\bar{e}_t w l + a + P_h h - \frac{\delta P_h h w}{A} + qh' + P_m(t, e, a', h', m'; \cdot, \Theta)m'$$

$$\tilde{m}' = \begin{cases} m' & \text{w.p. } \rho \\ 0 & \text{w.p. } 1 - \rho \end{cases} \quad F = \begin{cases} F^{issue} & \text{if } m = 0, m' > 0 \\ F^{ref} & \text{if } m > 0, m' > 0 \\ 0 & \text{if } m' = 0 \end{cases}$$

$$\delta' \sim F_\delta(\delta'), \quad e' \sim F_e(e'|e), \quad \Omega' = G(\Omega), \quad V^{ref}(T+1, \cdot) = 0$$

$$\left( \sum_{j=t}^T [\Pi_{i=t}^j \pi_i] \left[ \frac{\rho}{1+r} \right]^{j-t} \right) m' / P_h h' < LTV$$

# Market Clearing Conditions

1. Rental markets clears:

$$\sum_{t=1}^T \int g_r(t, e, a, h^o, m, \delta_h) d\Phi_t = \sum_{t=1}^T \int g_h(t, e, a, h^o, m, \delta_h) d\Phi_t$$

2. Housing market clear:

$$\sum_{t=1}^T \int g_r(t, e, a, h^o, m, \delta_h) d\Phi_t = \bar{H}$$

Go to equilibrium



## Government's Budget Balance

$$\underbrace{\int \mathbf{sm}(1 - g_{\text{default}}(j, e, a, h^o, m, \delta))d\Phi}_{\text{Subsidies}} + \underbrace{B}_{\text{Bailout}} = \underbrace{\int \tau \mathbf{c}d\Phi}_{\text{Taxes}}$$

$$B = \sum_{t=1}^T \left( \frac{\pi_j \rho}{1 + r_f} \right) \int \left[ \underbrace{g_{\text{def}}(\cdot) P_h (1 - \delta' - \bar{\delta}) \mathbf{H}'}_{\text{Default}} + \right. \\ \left. \underbrace{g_{\text{repay}}(\cdot) \left( \sum_{j=t+1}^T [\prod_{i=t+1}^j \pi_i] \left( \frac{\rho}{1+r} \right)^{j-(t+1)} \right)}_{\text{Renew mortgage}} (1-s) \mathbf{m}' + \right. \\ \left. \underbrace{(1 - g_{\text{def}}(\cdot))(1 - g_{\text{repay}}(\cdot))(\mathbf{m}' + P_m(\cdot) \mathbf{m}')}_{\text{Keep mortgage}} \right] - \mathbf{m}' P_m(\cdot) d\Phi$$

Go to equilibrium