

Syllabus for Econ 897

Summer 2016

Instructors:

Carlos Segura (Part I, email: cseg@sas.upenn.edu)

David Zarruk (Part II, email: davidza@sas.upenn.edu)

Joonbae Lee (Part III, email: joonbae@sas.upenn.edu)

Schedule:

July 11 - August 19, 2016: Monday, Tuesday, Wednesday, Thursday and Friday

Time: 10:00 AM-12:00 noon and 1:30 PM-3:30 PM each day

Final (Waiver) Exam: Monday, August 22, 2016

Location: TBA

Course Website: <http://economics.sas.upenn.edu/graduate-program/current-students/course-information/summer-math-camp>

Textbooks:

There are three textbooks for this course.

- For real analysis,

Real Mathematical Analysis by Charles C. Pugh, Springer 2010.

- For optimization,

A First Course in Optimization Theory by Rangarajan K. Sundaram, Cambridge 1996.

- For probability,

Statistical Inference by Casella and Berger (CB), Thomson Learning 2002. This one will be used again in ECON 705.

In addition, you might find some of the following books useful to supplement different parts of the lectures.

The classic *Principles of Mathematical Analysis* by Walter Rudin is a good reference. You can also have a look at the first chapters of *The Nature and Origins of Modern Mathematics: an*

Elementary Introduction by Andy McLennan (Available online: <http://cupid.economics.uq.edu.au/mclennan/NatureOrigins/natureorigins.html>). A short summary in optimization is also contained in the appendix of Mas-Colell, Whinston, Green (MWG): *Microeconomic Theory*. This is also a recommended book for Econ 701 and Econ 703.

If you are not familiar with the elementary set theory and structure of proofs, we strongly suggest that you read the section 1.1 in Pugh.

Exams and Homework:

There will be graded tests every Friday to cover the materials covered in the corresponding week. These two tests will be graded. Additionally, each instructor might give quizzes and will post problem sets. These might influence your grade as well. We strongly recommend you to try to solve the questions before you see solutions. The waiver exam is the final for all three parts of Econ 897. Your grade will be determined as the average of your grades for all three parts.

Outline of the Course:

Part I. Weeks 1 and 2

- (a) The Real Numbers
 - i. Properties of \mathbb{R} . (Pugh, §1.2)
 - ii. Euclidean Space (Pugh, §1.3)
 - iii. Functions and Cardinality (Pugh, §1.4)
 - iv. The Skeleton of Calculus (Pugh, §1.6)
- (b) Metric Spaces and Very Basic Topology (Pugh, §2.1)
 - i. Metrics, Sequences, Convergence and Limits
 - ii. Closed and Open Sets, Accumulation Points, Boundaries, Closure
 - iii. Topological Continuity and Homeomorphisms
 - iv. Cauchy Sequences and Completeness
 - v. Topological and Sequential Limits
- (c) Compactness and Connectedness

- i. Sequential Compactness (Pugh, §2.2)
- ii. Heine-Borel and Weierstrass Theorems (Pugh, §2.2)
- iii. Extreme Value Theorem (Pugh, §2.2)
- iv. Connectedness (Pugh, §2.3)
- v. Coverings (Pugh, §2.4)
- vi. Continuity of Correspondences (Sundaram, §9.1; MWG, Appendix M.H)

Part II. Weeks 3 and 4

- (a) Differentiation: (Pugh, §3.1)
 - i. Definition and basic properties
 - ii. Mean value theorem and inverse function theorem
 - iii. Higher order derivatives and Taylor's theorem
- (b) Linear Algebra:
 - i. Determinants (Sundaram, §1.3; Pugh, Chapter 5 Appendix E)
 - ii. Vector spaces and linear mappings (Sundaram, Appendix C.1; Pugh §5.1)
 - iii. Inner product and orthogonality (Sundaram, Appendix C.2)
- (c) Analysis in \mathbb{R}^n :
 - i. Partial and total derivatives (Pugh, §5.2)
 - ii. Higher derivatives (Pugh, §5.3)
 - iii. Implicit and inverse functions (Pugh, §5.5)
 - iv. Convex Sets and Separating Hyperplane Theorems (MWG, Appendix M.G)
 - v. Concavity and Quasi-Concavity (Sundaram, §7.1-7.2 and §8.1-8.3)

Part III. Weeks 5 and 6

- (a) Optimization Theory in \mathbb{R}^n :
 - i. Definitions and the Basic Setup (Sundaram, §2.1-2.4)
 - ii. Characterization of Optimal Solutions
 - Unconstrained Optimization Problems (Sundaram, §4.1-4.6; MWG, Appendix M.J)
 - Equality Constraints and the Theorem of Lagrange (Sundaram, §5.1-5.7; MWG, Appendix M.K)

- Inequality Constraints and the Theorem of Kuhn-Tucker (Sundaram, §6.1-6.5; MWG, Appendix M.K)
 - Concavity and Quasi-Concavity in Optimization (Sundaram, §7.3-7.4 and §8.3-8.5)
- iii. Sensitivity
- The Maximum Theorem (Sundaram, §9.2)
 - The Envelope Theorems (MWG, Appendix M.L)
- (b) Brief Review of Basic Probability Theory
- i. Basics of Probability Theory (CB, §1.1-1.2)
- ii. Random Variables (CB, §1.4)
- iii. Transformations, Expectations and Moments (CB, §2.1-2.2)
- iv. Common Families of Distributions
- Discrete Distributions (CB, §3.2)
 - Continuous Distributions (CB, §3.3)
- v. Multiple Random Variables
- Joint and Marginal Distributions (CB, §4.1)
 - Conditional Distributions and Independence (CB, §4.2)
 - Multidimensional Transformations (CB, §4.3)
- vi. Convergence Concepts
- Convergence in Probability (CB, §5.5.1)
 - Convergence in Distributions (CB, §5.5.3)