

ECON 897 Test (Week 3)
July 31, 2015

Important: This is a closed-book test. No books or lecture notes are permitted. You have **120** minutes to complete the test. Answer all questions. You can use all the results covered in class, but please make sure the conditions are satisfied. Write your name on each blue book and label each question clearly. Write legibly. Good luck!

1. (20 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be twice differentiable. Suppose there exists $\epsilon > 0$ such that $f''(x) > \epsilon$ for all $x \in \mathbb{R}$. Show that $f'(x) = 0$ for some $x \in \mathbb{R}$.
2. (20 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be twice continuously differentiable. Assume there is a $c \in (a, b)$ such that $f'(c) = 0$ and $f''(c) < 0$. Show that f has a local maximum at c .
3. (20 points) Suppose that $f : (a, b) \rightarrow \mathbb{R}$ is differentiable and f' is bounded. If $\{x_n\}$ is a sequence on (a, b) and $x_n \rightarrow a$, then $f(x_n)$ converges.
4. (20 points) State whether the following are linear subspaces, and prove your answer:
 - (a) Let $W_n = \{f(x) \in P(F) \mid f(x) = 0 \text{ or } f(x) \text{ has degree exactly equal to } n > 1\}$. Is W_n a subspace of $P(F)$, where $P(F)$ is the space of polynomials?
 - (b) Let $A = \{(a_1, a_2, a_3) \in \mathbb{R}^3 \mid a_1 = a_3 + 2\}$. Is A a subspace of \mathbb{R}^3 ?
5. (20 points) Let $A \in M_{n \times n}$, such that A^{-1} exists. Prove that the columns of A form a basis for \mathbb{R}^n .

For the following problem, you cannot use the results in the exercises:

6. (20 points) If a is an $n \times 1$ vector, and b is a $1 \times m$ vector, prove that ab is an $n \times m$ matrix of rank at most equal to one.